

5-3 Monomial Factors of Polynomials

Objective: To divide polynomials by monomials and to find monomial factors of polynomials.

Vocabulary

Divisible One polynomial is (evenly) divisible by another polynomial if the quotient is also a polynomial. Example 1b shows that $27uv - 36v$ is divisible by $9v$.

Greatest monomial factor of a polynomial The GCF of the terms of the polynomial. In Example 3, the GCF of $3x^2 + 12x$ is $3x$.

Example 1 Divide: a. $\frac{6m + 36}{6}$ b. $\frac{27uv - 36v}{9v}$

Solution Divide each term of the polynomial by the monomial. Then add the results.

$$\begin{aligned} \text{a. } \frac{6m + 36}{6} &= \frac{6m}{6} + \frac{36}{6} & \text{b. } \frac{27uv - 36v}{9v} &= \frac{27uv}{9v} - \frac{36v}{9v} \\ &= m + 6 & &= 3u - 4 \end{aligned}$$

Example 2 Divide: $\frac{2x^4 - 8x^3y + 4x^2y^2}{-2x^2}$

Solution $\frac{2x^4 - 8x^3y + 4x^2y^2}{-2x^2} = \frac{2x^4}{-2x^2} - \frac{8x^3y}{-2x^2} + \frac{4x^2y^2}{-2x^2} = -x^2 + 4xy - 2y^2$

Divide. Assume that no denominator equals zero.

1. $\frac{4a + 12}{4}$
2. $\frac{10a - 15}{5}$
3. $\frac{20n - 16}{4}$
4. $\frac{6x + 9y + 12}{3}$
5. $\frac{2m - 4n + 6}{2}$
6. $\frac{x^3 - 4x^2 + 6x}{x}$
7. $\frac{8xy - 12x^2}{4x}$
8. $\frac{5a - 10a^2 - 15a^3}{5a}$
9. $\frac{12y - 18y^2 - 6y^3}{6y}$
10. $\frac{4x^2 - 12x - 8}{4}$
11. $\frac{27y^4 + 18y^3 - 36y^2}{9y^2}$
12. $\frac{6u^3 + 4u^2 - 2u}{2u}$
13. $\frac{12r^4 - 9r^3 - 6r^2}{-3r^2}$
14. $\frac{5m^3 + 8m^4 - 3m^5}{-m^3}$
15. $\frac{xy^3 + x^3y}{xy}$
16. $\frac{8ab^2 - 12a^2b}{4ab}$

5-3 Monomial Factors of Polynomials (continued)**Example 3** Factor $3x^2 + 12x$ **Solution**1. The greatest monomial factor of $3x^2 + 12x$ is $3x$.2. Divide to find the other factor:
$$\frac{3x^2 + 12x}{3x} = \frac{3x^2}{3x} + \frac{12x}{3x}$$
$$= x + 4$$
3. $3x^2 + 12x = 3x(x + 4)$ **Example 4** Factor $6x^5 - 4x^3 + 8x$ **Solution**1. The greatest monomial factor of $6x^5 - 4x^3 + 8x$ is $2x$.2. Divide to find the other factor:
$$\frac{6x^5 - 4x^3 + 8x}{2x} = \frac{6x^5}{2x} - \frac{4x^3}{2x} + \frac{8x}{2x}$$
$$= 3x^4 - 2x^2 + 4$$
3. $6x^5 - 4x^3 + 8x = 2x(3x^4 - 2x^2 + 4)$ **Factor.**

17. $21a^3 - 14a^2$

18. $4x^3 + 32x$

19. $9x^2 + 36x$

20. $21c^3 - 14c$

21. $10a - 35b + 15$

22. $16x - 12y + 24$

23. $8p - 4q + 12$

24. $3x - 6y + 12$

25. $9x - 6y + 36$

26. $15a - 20b + 10$

27. $3a^3 + 6a^2 - 12a$

28. $10x^3 - 5x^2 + 20x$

29. $5y^3 - 10y^2 + 15y$

30. $18x^3 - 6x^2 + 24x$

31. $8ab^2 - 12a^2b$

32. $3a^2b^2 + 18ab$

33. $6y^3 - 24y^2 - 12y$

34. $20y^4 + 35y^3 + 15y^2$

Mixed Review Exercises**Simplify.**

1. $6n^2\left(\frac{1}{6}n^2\right)$

2. $8x^2\left(\frac{3}{4}x^3\right)$

3. $3a^2 - 6ac^2 + 4a^2 - 5ac^2$

4. $\frac{5x^3y}{10x^2y^2}$

5. $24 \div \left(-\frac{1}{3}\right)$

6. $\frac{(3a^2)^3}{a^4}$

7. $(3a)^4$

8. $6(3^2 - 1) + 2^3$

9. $(x - 1)(x^2 + 2x + 3)$

10. $(m - 3)(m + 4)$

11. $(3a + 2)(5a - 3)$

12. $(6p - q)(2p - 3q)$